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Calculation of Transient Boiling Flow in Channels

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A new method of calculating transient two-phase flow is presented, based on a modification of the implicit multifield (IMF) method, and is applied to channel boiling. The two-phase two-fluid approach allows flexibility in physically modelling different flow regimes. The numerical finite difference method treats the vapor phase more implicitly than does IMF. Detailed analysis of the incremental pressure iterations in the implicit cycle of each time step shows that the new pressure profile satisfies approximately a discrete elliptic boundary value problem, with adjacent nodes strongly coupled if the time step is large. By accounting for this, order-of-magnitude gains in computation time have been achieved.

INTRODUCTION

The range of hypothetical accidents posed for sodium-cooled fast reactors can lead to channel boiling transients which differ widely in nature. On one hand, a rapid pump coastdown or flow blockage incident may lead to a very rapid boiling transient, voiding a channel in a few seconds or less. On the other hand, a very slow pump coastdown, accompanied perhaps by a decrease in reactor power, can lead to boiling transients initially so slow as to be almost a succession of steady states. It is in order to treat effectively this range of possibilities that the present calculation method was developed.

Two-phase flow models describe boiling liquid flow by conservation equations for mass, momentum, and energy. The one-dimensional two-phase two-fluid model includes conservation equations in the axial (channel length) direction for each of the phases considered as a separate fluid, with terms describing the exchange of mass, energy, and momentum between the phases and with the subchannel walls. This general model does not assume any particular flow regime; information about the flow regime is reflected in the various exchange terms. Thus it is possible to treat a variety of flows with the same basic method, by modifying exchange terms. Indeed another possible application of the present method is to the physically different problem of blowdown from a ruptured PWR coolant pipe [1]. A further attraction of two-phase flow modelling is the possibility of treating multiple connected subchannels using transmission coefficients between subchannels (cf. [2]), and of full two-dimensional calculation of large pipes.

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Two-phase flow calculation methods have long been applied to channel flow problems in sodium and water cooled reactors, for example, in the codes FLICA [2, 3] and FLINA [4]. These codes use a numerical method which we call *directed*: in each channel, one begins at one end, usually the inlet, and integrates one axial level at a time toward the other end. If the boundary conditions involve the outlet pressure, a shooting method is used to find, by successive trials, the inlet flow conditions which when integrated match the desired outlet pressure. Directed methods, although well suited for slow to moderately fast transients, are limited in principle by the use of the shooting method, which requires that during a time step Δt, any perturbation at the outlet has time to propagate and be felt at the inlet. This means a lower limit to the time step and/or an upper limit on the extent of the two-phase region.

To avoid any such limitation, we shall use what we call a *parallel* method, wherein new time values are found by advancing all the space points simultaneously; boundary conditions at both ends of the channel can be satisfied without shooting. Examples of parallel methods already in use are the implicit multifield (IMF) technique [5, 6] and, for single fluids, the method of characteristics and the method of lines. Parallel methods are also more susceptible of generalization to two or three space dimensions.

Another trait which distinguishes various methods is the degree to which they are implicit. Implicit here refers to any expression in the discretized equations involving values at the new time $t + \Delta t$, as opposed to explicit terms which use only values at the old time. Of course each term which is treated implicitly may add to the complexity of the problem: it means another term linking the unknown values at $t + \Delta t$. However, implicit methods have the important advantage of being stable for larger time steps. Directed methods can be made highly implicit because the unknowns are considered at only one node at a time. (This will be discussed in detail below). The consequent stability for large Δt, together with the lower limit, explain why implict directed methods are well suited for relatively slow transients.

Parallel methods are more difficult to implement efficiently if cast in totally implicit form; existing two-fluid methods such as ICE [7] and IMF use a partially explicit treatment. The parallel property removes any lower limit on Δt, but the greater explicitness brings definite upper limits. One usually tries to treat implicitly at least those terms having to do with signal propagation at sonic velocity c, so as to avoid the limitation $\Delta t \leqslant \Delta z/c$. However, phenomena displaced with a fluid at velocity u are treated explicitly, resulting in a bound of the type $\Delta t \lesssim \Delta z/u$ (or perhaps a specific fraction thereof).

The ideal method would be a fully implicit parallel method, which would in principle have neither an upper nor a lower limit to the time step size. Such a method could take long time steps when the flow evolves slowly, and small steps to study abrupt local perturbations. In the present work, our goal has been to develop a parallel method sufficiently implicit to bypass time step limits related to the sonic velocity c and to the vapor velocity u_v, leaving only the limit $\Delta t < \frac{1}{4}\Delta z/u_l$. For the application to low-pressure channel boiling this can be quite helpful, as the vapor may travel at a velocity many times that of the liquid.

In order to implement our method, we have found it necessary to analyze more carefully the implicit phase of the time step advancement. As in the IMF technique, we determine the new pressures by iteration. As Δt is allowed to become larger than $\Delta z/c$, the pressures at neighboring nodes become strongly interdependent, leading to a problem of elliptic type. Our analysis shows how to take this into account, so that our procedure is highly efficient for $\Delta t > \Delta z/c$.

The interdependence of nodes can also be thought of in terminology of stiff systems of differential equations (e.g., [8, 9])[1]. Spatial discretization can reduce any initial value problem for a partial differential equation to a system of ordinary differential equations [9]; the resulting system of ordinary differential equations is called stiff if its spectrum contains a wide spread of eigenvalues, and this in turn can result from strong spatial coupling in the original partial differential equations. The method of lines [9], if applied to two fluid models, would be a fully implicit parallel method. Our procedure accounts for the spatial coupling due to sonic phenomena (but not for coupling due to liquid convection) in a simpler way, based on physical properties of the flow equations, generalizing techniques already in use for single-fluid compressible flow (e.g., [7, p. 203]).

Before explaining our parallel method, which we have labeled XI (explicit–implicit), we shall review some aspects of the two-phase two-fluid flow model. The XI method focuses on that part of the channel in two-phase flow. We shall also describe the application to sodium channel boiling transients, some of the additional techniques we have developed for this application, and some sample calculations carried out. Substantial further development is planned to fit these methods into a more realistic context.

We shall try to make this presentation self-contained, but the papers of Harlow and Amsden [5–7] are nevertheless fundamental and highly recommended.

Two-Fluid Flow Model

Consider a one-component two-phase fluid—a liquid and its vapor—flowing in a channel of constant cross section and possibly heated. We shall treat the flow in a one-dimensional approximation, so that at each point along the channel we define flow variables which represent values averaged over a small volume cutting across the channel, and over a short time interval; the numerous bubbles and droplets are approximated as interpenetrating continua. Thus at each point we introduce a macroscopic pressure P, a void fraction (or vapor volume fraction) α, average liquid and vapor velocities u_l and u_v, and average liquid and vapor enthalpies h_l and h_v. In particular, for the case of annular mist flow in the sodium application, the liquid wetting the fuel pin is averaged with the entrained drops. The basic equations

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[1] The author owes this remark to one of the referees.

describe the conservation of the mass, energy, and momentum of each of the phases. For the mass we have

$$\frac{\partial \alpha \rho_{\mathrm{v}}}{\partial t} + \nabla \cdot (\alpha \rho_{\mathrm{v}} u_{\mathrm{v}}) = S_{\mathrm{e}} - S_{\mathrm{c}}\,, \tag{1}$$

$$\frac{\partial (1-\alpha)\,\rho_l}{\partial t} + \nabla \cdot ((1-\alpha)\,\rho_l u_l) = S_{\mathrm{c}} - S_{\mathrm{e}}\,, \tag{2}$$

where S_{e} and S_{c} represent mass transfer by evaporation and condensation, respectively. For the conservation of momentum we pose

$$\frac{\partial \alpha \rho_{\mathrm{v}} u_{\mathrm{v}}}{\partial t} + \nabla \cdot (\alpha \rho_{\mathrm{v}} u_{\mathrm{v}}^2) + \alpha \nabla P = u_l S_{\mathrm{e}} - u_{\mathrm{v}} S_{\mathrm{c}} - K(u_{\mathrm{v}} - u_l) - \alpha \rho_{\mathrm{v}} g, \tag{3}$$

$$\begin{aligned} &\frac{\partial (1-\alpha)\,\rho_l u_l}{\partial t} + \nabla \cdot {}'(1-\alpha)\,\rho_l u_l^2) + (1-\alpha)\,\nabla P \\ &\quad = u_{\mathrm{v}} S_{\mathrm{c}} - u_l S_{\mathrm{e}} + K(u_{\mathrm{v}} - u_l) - (1-\alpha)\,\rho_l g - F. \end{aligned} \qquad {}'4)$$

The term $K(u_{\mathrm{v}} - u_l)$ represents the momentum exchange by friction between the vapor and the liquid. We have assumed that the wall friction law F acts only on the liquid, and that the channel is oriented vertically upward. For the momentum exchange associated with mass transfer, we suppose that all liquid which is evaporated off will enter the vapor flow with velocity u_l, while condensing vapor will form into droplets of initial velocity u_{v}. In particular, $u_{\mathrm{v}} = u_l$ at a boiling front.

For the energy balance we write

$$\frac{\partial \alpha \rho_{\mathrm{v}} h_{\mathrm{v}}}{\partial t} + \nabla \cdot (\alpha \rho_{\mathrm{v}} h_{\mathrm{v}} u_{\mathrm{v}}) - \alpha \frac{\partial P}{\partial t} - \alpha u_{\mathrm{v}} \cdot \nabla P = R + q_{\mathrm{v}}\,,$$

$$\frac{\partial (1-\alpha)\,\rho_l h_l}{\partial t} + \nabla \cdot ((1-\alpha)\,\rho_l h_l u_l) - (1-\alpha)\frac{\partial P}{\partial t} - (1-\alpha)\,u_l \cdot \nabla P = -R + q_l\,,$$

where $q = q_{\mathrm{v}} + q_l$ is an external source of energy by heating (or sink by cooling), with q_{v} acting on the vapor and q_l on the liquid. We have neglected conduction within each fluid, as well as the energy dissipated by friction. R represents energy exchange between the phases. Note that the terms are equal and opposite, but the same is not true of the heat transfer associated with mass transfer, where the latent heat intervenes.

We shall set forth some simplifying physical assumptions used in implementing this method; other variants are also possible, and we are not necessarily advocating these choices, but simply fixing ideas.

First we neglect the compressibility of the liquid, so that $\rho_l = \rho_l(h_l)$. Second we suppose that the vapor always remains at saturation, so that $\rho_{\mathrm{v}} = \rho_{\mathrm{v}}(P)$ and $h_{\mathrm{v}} = h_{\mathrm{v}}(P)$. Furthermore, we assume boiling only ($S = S_{\mathrm{e}}$) and we introduce a relaxation model for the energy $Q(= -R - h_l S_{\mathrm{e}})$ given up by the liquid as boiling occurs:

$$Q = C(T_{\mathrm{v}} - T_l) = \tau^{-1}(1-\alpha)\,\rho_l(h_{l,\mathrm{sat}}(P) - h_l).$$

Thus by transforming the above equations we obtain

$$\alpha\rho_V\left[\frac{\partial h_V}{\partial t} + u_V\cdot\nabla h_V\right] - \alpha\left[\frac{\partial P}{\partial t} + u_V\cdot\nabla P\right] = -Q + S(h_V - h_l) + q_V\,, \qquad (5)$$

$$(1-\alpha)\,\rho_l\left[\frac{\partial h_l}{\partial t} + u_l\cdot\nabla h_l\right] - (1-\alpha)\left[\frac{\partial P}{\partial t} + u_l\cdot\nabla P\right] = Q + q_l\,. \qquad (6)$$

Now the liquid energy equation relates the liquid disequilibrium to a relaxation process taking place in the moving liquid. The assumption $h_V = h_V(P)$ changes the use of (5): it now expresses a relation between the vapor formation rate S_e and the external heating $q_l + q_V$, with corrections for the axial variation of vapor energy (left side of (5)) and liquid energy (left side of (6), via Q in (5)).

We adopt the following form for the momentum exchange coefficient K (cf. [6]):

$$K = \tfrac{3}{8}\alpha(1-\alpha)(1/r^2)[12\mu + C_D r\rho\,|\,u_V - u_l\,|],$$

where it remains to specify, for a given flow regime, whether r, μ, and ρ are associated with one phase or the other or some sort of average. In the case of isolated non-deformable spheres of vapor with radius r_V and moving in an infinite liquid medium, one should have $r = r_V$, $\mu = \mu_l$, and $\rho = \rho_l$ with $C_D = 0.5$; for liquid spheres in a continuous vapor medium, the phase subscripts are of course reversed. Our experience with channel boiling calculation allows a few remarks. At low pressures, the flow regime in long narrow subchannels is likely to be annular or turbulent slug-annular, with vapor cavities whose transverse dimensions are on the order of magnitude of the subchannel diameter. These vapor regions undergo severe deformation, with liquid perhaps breaking up, forming detached chunks or drops which later coalesce. In any case, whether the radius r is associated with the vapor cavities or with the liquid agglomereations, no value larger than the hydraulic diameter has any direct physical interpretation, and we would expect a radius on that scale. Now from our experience we can say that neither the vapor nor the liquid viscosity term in K above would suffice to explain the observed momentum transfer, and the drag (or interphase friction) term must dominate in this flow regime. But at low pressures, the phase densities differ by three orders of magnitude, so again from experience we can say that $\rho = \rho_V$ comes much closer to giving the observed momentum transfer than $\rho = \rho_l$. This apparently means that the relative motion of the phases at low pressures is determined much more by the ease with which liquid chunks pass through vapor by bubble deformation, than by the resistance encountered by a bubble pushing through liquid. Of course, detailed knowledge of K will require extensive experimental study of two-phase flow regimes in the light of the two-fluid model.

Naturally, K may be even more complicated than the expression above, or have a considerably different form, and the energy transfer constant τ and the wall friction law F may also be functions of the flow variables. A serious effort will be needed to adjust these terms to experiments. It should be remembered that while microscopic models may be suggestive for the form of K and τ, the two-fluid model uses variables which are space-time averages, as indeed are many of the experimentally measured values. However, we shall not, and should not need to, account specifically for these possible complications in describing the general numerical method which follows.

XI METHOD

The two-phase parallel method is applied on a Eulerian mesh over a length of channel where the flow is entirely two-phase ($0 < \alpha < 1$). The mesh consists of cells of width Δz. The state variables α, P, and h_l will be edge-centered, that is, discrete values will be sought at the edge of each cell. The velocities will be cell-centered. Auxiliary values—cell-centered state variables and edge-centered velocities—will be found as needed by linear interpolation. (An exception is the liquid volume fraction $(1 - \alpha)$, for which the inverse will be linearly interpolated because for low qualities $\alpha/(1 - \alpha)$ is more nearly linear than α).

First the terms of the six equations (1)–(6) should be classified according to whether they will be treated explicitly or implicitly. As indicated above, our goal is to construct a method sufficiently implicit that the time step will not be limited by c or u_v, but only by u_l. In order to treat sonic propagation implicitly, we follow the ICE and IMF methods and select the pressure gradient term in the momentum equations for implicit differencing, as well as the vapor mass equation (which contains the primary manifestation of vapor compressibility). On the other hand, by accepting time step limitations relating to the liquid velocity, we can treat the liquid mass equation explicitly. As we shall see this will in general require that an artificial mass diffusion term be added to the liquid mass equation in order to guarantee stability [7, 10]. We may also difference the liquid momentum gradient in (4) explicitly.

Our main change from the IMF differencing scheme is that, in addition to treating the momentum coupling implicitly, we treat vapor motion fully implicitly; that is, the vapor momentum gradient in Eq. (3) is differenced at time $t + \Delta t$, along with the pressure gradient. This permits time steps larger than $\Delta z/u_v$.

Finally, we treat the energy and mass transfer explicitly. Although this is not an uncommon choice, it should be noted that theoretically this could conceal another limitation on the time step size. For example, if the liquid enthalpy relaxation equation is simplified by omitting the relatively small pressure terms,

$$\frac{\partial h_l}{\partial t} + u_l \cdot \nabla h_l = \frac{h_{l,\text{sat}} - h_l}{\tau} + q_l,$$

it is clear that an explicit difference method would be stable only for $\Delta t < \tau$. It turns out that in many cases this limitation is not serious, since the energy coupling between the phases is not so strong as the momentum coupling. However, one should be aware of the possible restriction.

Let us consider the finite difference equations in detail. The space-time mesh is shown in Fig. 1. To avoid overloading equations with subscripts, we have chosen

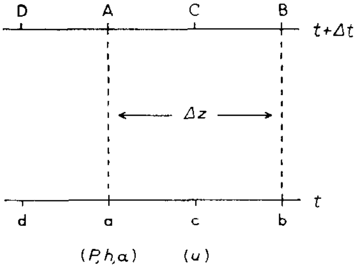

FIG. 1. Space-time mesh and position of variables.

a typical cell whose edge nodes are denoted at time t by a, b and by A, B at time $t + \Delta t$. The included cell-centered node is c (resp. C), and a neighboring cell-centered node is indicated by d (resp. D).

With an explicit/implicit method, each time step calculation falls into two phases: the first, explicit phase involves calculations which are done only once in each time step for each node; the second, implicit part is an iterative procedure. Starting with the energy balance equations, we have the explicit difference equation (from (6) divided by $(1 - \alpha)\rho_l$):

$$\begin{aligned}(h_{lB} - h_{lb})/\Delta t + u_{lc}(h_{lb} - h_{la})/\Delta z - (P_B - P_b)/(\rho_{lb}\Delta t) - u_{lc}(P_b - P_a)/(\rho_{lb}\Delta z)\\ = (h_{l,\mathrm{sat}}(P_b) - h_{lb})/\tau_b + q_{lb}/((1 - \alpha_b)\,\rho_{lb}).\end{aligned} \tag{7}$$

The notation is slightly inaccurate, since the difference $(P_B - P_b)$ uses not the definitive value of P_B but an estimate for P_B furnished by extrapolation from previous time steps; this amounts to an explicit treatment. This presents no problem since the term is not significant compared to the others. We also note that the above difference equation is for the case $u_{lc} \geqslant 0$; if $u_{lc} \leqslant 0$, we interchange the roles of a and b, and replace B by A, so that h_l is always calculated in the physically appropriate direction, with the flow. Difference equation (7) gives the new liquid enthalpy at each point.

Next consider the difference equation of vapor energy balance from (5):

$$\begin{aligned}&\alpha_b\rho_{\mathrm{v}b}[(h_{\mathrm{v}B} - h_{\mathrm{v}b})/\Delta t + u_{\mathrm{v}c}(h_{\mathrm{v}b} - h_{\mathrm{v}a})/\Delta z]\\ &\quad - \alpha_b[(P_B - P_b)/\Delta t + u_{\mathrm{v}c}(P_b - P_a)/\Delta z]\\ &\quad + (1 - \alpha_b)\,\rho_{lb}[h_{l,\mathrm{sat}}(P_b) - h_{lb}]/\tau_b + S_B(h_{\mathrm{v}B} - h_{lB}) = q_{\mathrm{v}b}\,.\end{aligned} \tag{8}$$

The previous remarks concerning P_B and the sign of u_{v} apply. This equation gives the new net mass transfer rate $S_B = (S_{\mathrm{e}} - S_{\mathrm{c}})_B$.

Now the liquid mass can be advanced explicitly using the difference equation:

$$\begin{aligned}&[(1 - \alpha_B)\,\rho_{lB} - (1 - \alpha_b)\,\rho_{lb}]/\Delta t\\ &\quad + [(1 - \alpha_d)\,\rho_{ld}u_{ld} - (1 - \alpha_c)\,\rho_{lc}u_{lc}]/\Delta z = -S_B - \lambda_b\,.\end{aligned} \tag{9}$$

Here λ is the artificial mass diffusion included to stabilize the calculation (see below). Since ρ_{lB} is a function of h_{lB} it is now known, so Eq. (9) gives the new void fraction α_B.

There remain some further quantities which can profitably be calculated before entering the iterative phase. They include the momentum terms corresponding to external forces (gravity, wall friction) and to the gradient of liquid momentum, as well as K_1 and K_2, where $K = K_1 + K_2 \mid u_{\text{v}} - u_l \mid$, in the momentum exchange. This completes the explicit phase of the time step.

During the implicit phase, the remaining equations of motion are used to determine the new pressure and velocity profiles. This requires an iterative procedure, for which we introduce the following notation: values at the beginning of each iteration shall be indicated by a circumflex (^), values resulting from the iteration shall be denoted by a tilda (~). (Beginning values for the first iteration can be had by linear extrapolation from previous time steps, for example; since the convergence of the iterations is very rapid, a good starting point is helpful but not crucial.) The iteration procedure can be outlined as follows:

(i) Starting from an estimated pressure profile $\hat{P}_A$, $\hat{P}_B$,... we solve the two momentum conservation equations for new estimates of the velocities $\tilde{u}_{lC}$, $\tilde{u}_{lD}$,... and $\tilde{u}_{\text{v}C}$, $\tilde{u}_{\text{v}D}$,... .

(ii) From the new velocities we calculate the conservation of vapor mass at each point, finding the balances not zero but equal to remainders $\tilde{r}_A$, $\tilde{r}_B$,... .

(iii) These remainders determine increments δP_A, δP_B,... for the estimated pressures so that $\tilde{P}_A = \hat{P}_A + \delta P_A$, etc. If the increments are not smaller than a specified error ϵ, the new pressure profile $\tilde{P}_A$, $\tilde{P}_B$,... is used to recommence the iteration.

In the third step we introduce a new method of calculating the increments, which is essential to success with $\Delta t \gg \Delta z/c$. We now explain these three steps further.

Step (i) uses the momentum equations to find new velocities. Actually we use modifications of Eqs. (3) and (4). Instead of the liquid momentum equation we use a total momentum equation which is just the sum of (3) and (4), and offers the convenience that all the momentum transfer terms drop out. And instead of the vapor momentum equation (3), we use a vapor acceleration equation obtained by substituting (1) into (3). The finite difference equation for the total momentum is:

$$\begin{aligned}
&[(\alpha_C \hat{\rho}_{\text{v}C} \tilde{u}_{\text{v}C} + (1 - \alpha_C)\, \rho_{lC} \tilde{u}_{lC}) - (\alpha_c \rho_{\text{v}c} u_{\text{v}c} + (1 - \alpha_c)\, \rho_{lc} u_{lc})]/\Delta t \\
&\quad + [(\alpha_b \rho_{\text{v}b} u_{\text{v}b}^2 + (1 - \alpha_b)\, \rho_{lb} u_{lb}^2) - (\alpha_a \rho_{\text{v}a} u_{\text{v}a}^2 + (1 - \alpha_a)\, \rho_{la} u_{la}^2)]/\Delta z \\
&\quad + (\hat{P}_B - \hat{P}_A)/\Delta z = -(\alpha_c \rho_{\text{v}c} + (1 - \alpha_c)\, \rho_{lc})\, g - F_c\,. \qquad (10)
\end{aligned}$$

Note that vapor momentum convection is treated explicitly here, but the effect is found to be negligible in the total momentum equation because $\rho_l \gg \rho_{\text{v}}$. All terms with neither ^ nor ~ are known from either the previous time step or the explicit phase, so as far as the implicit phase is concerned we have

$$(\alpha_C \hat{\rho}_{\text{v}C} \tilde{u}_{\text{v}C} + (1 - \alpha_C)\, \rho_{lC} \tilde{u}_{lC})/\Delta t + (\hat{P}_B - \hat{P}_A)/\Delta z = \cdots .$$

The finite difference equation for the vapor acceleration is:

$$\begin{aligned}\alpha_A\hat{\rho}_{VA}[(\tilde{u}_{VC} - u_{Vc})/\Delta t + \tilde{u}_{VC}(\tilde{u}_{VC} - \tilde{u}_{VD})/\Delta z]\\ = -\alpha_C(\hat{P}_B - \hat{P}_A)/\Delta z - \alpha_B\hat{\rho}_{VB}\, g - (S_C + K_{1C})(\tilde{u}_{VC} - \tilde{u}_{lC})\\ - K_{2C}\,|\,\tilde{u}_{VC} - \tilde{u}_{lC}\,|(\tilde{u}_{VC} - \tilde{u}_{lC}).\end{aligned} \tag{11}$$

Certain terms are offset from precise space centering for convenience. In particular, the velocity gradient involves $\tilde{u}_{VD}$ and apparently couples the equations for new velocities at all the nodes. Our solution procedure avoids possible complication by starting at one end where the velocity is a boundary condition ($u_V = u_l = 0$ or u_l and u_V known inflow, for example), and moving in sequence, so that in solving for $\tilde{u}_{VC}$, $\tilde{u}_{VD}$ is always known. Despite the practical advantage of this procedure, there are two possible objections. One is that the generalization to more than one space dimension is not obvious. Another is that the sequential method looks suspiciously like a directed, not a parallel method. Against the second criticism there are two defenses: although the method is organized in one direction, there is no shooting to match the outlet pressure; moreover, in practice the velocity gradient term, while not negligible is less than the pressure gradient and momentum exchange terms which are properly centered, and we have encountered no difficulty in calculating sonic propagation phenomena with $\Delta t < \Delta z/c$.

Step (i) makes use of these two momentum difference equations in the following way. With an initial pressure profile $\hat{P}_A$, $\hat{P}_B$,... and the corresponding vapor densities $\hat{\rho}_{VA}$, $\hat{\rho}_{VB}$,..., the total momentum equation gives a simple relation between $\tilde{u}_{lC}$ and $\tilde{u}_{VC}$. This relation may be substituted into the vapor motion equation to eliminate $\tilde{u}_{lC}$, leaving a quadratic equation for $\tilde{u}_{VC}$. (Here we see it is no burden to treat the quatratic K_2 term implicitly, since the quadratic degree is already imposed by the velocity gradient term). The correct root $\tilde{u}_{VC}$ is easily determined provided that $(\tilde{u}_{VC} - u_{Vc})/u_{Vc} \ll 1$. If this condition failed, a smaller Δt would be necessary, but we have never encountered this. (The method has not been tried for a calculation starting from fluids at rest.) Now returning to the total momentum equation gives $\tilde{u}_{lC}$.

Step (ii) uses the vapor mass finite difference equation

$$(\alpha_A\hat{\rho}_{VA} - \alpha_a\rho_{Va})/\Delta t + (\alpha_C\hat{\rho}_{VC}\tilde{u}_{VC} - \alpha_D\hat{\rho}_{VD}\tilde{u}_{VD})/\Delta z - S_A = \tilde{r}_A\,. \tag{12}$$

From the remainders so calculated, we shall find increments δP to change the pressure profiles $\hat{P}_A$, $\hat{P}_B$,... to an improved profile $\tilde{P}_A$, $\tilde{P}_B$,... .

The procedure for doing this, step (iii), is based on the variations in vapor densities and velocities resulting from a given change in the pressures. Starting from Eqs. (12) we calculate

$$\delta r_A \simeq \frac{\alpha_A}{\Delta t}\frac{d\rho_V}{dP}\,\delta P_A + \frac{\alpha_C\rho_{VC}}{\Delta z}\,\delta u_{VC} - \frac{\alpha_D\rho_{VD}}{\Delta z}\,\delta u_{VD}\,.$$

We have dropped the variations of ρ_{VC} and ρ_{VD}, which tend to cancel each other out, but variations of $u_{\mathrm{v}C}$ are essential. From the momentum equations, we find, for example,

$$\delta u_{\mathrm{v}C} \frac{\alpha_A \rho_{\mathrm{v}A}}{\Delta t} + \frac{\alpha_A \rho_{\mathrm{v}A}}{\Delta z} u_{\mathrm{v}C} + K'_C \simeq \frac{\alpha_C}{\Delta z} \delta P_A - \frac{\alpha_C}{\Delta z} \delta P_B + K_C \, \delta u_{lC}$$

where

$$K'_C = S_C + K_{1C} + 2K_{2C}(u_{\mathrm{v}C} - u_{lC})$$

and

$$\alpha_C \rho_{\mathrm{v}C} \delta u_{\mathrm{v}C} + (1 - \alpha_C) \, \rho_{lC} \delta u_{lC} \simeq \frac{\Delta t}{\Delta z} (\delta P_A - \delta P_B).$$

Any change of P_A will result in changes in the velocities on either side of point A, as well as a change in the density $\rho_{\mathrm{v}A}$; the changes in velocities will in turn cause a change in the remainder r not only at A but at the points on either side of A (B and the unnamed point to the left of D). Note here the suitability of having chosen velocities between the pressure nodes.

Combining these equations, and reverting to index subscripts to identify space points, so that $A = j$, $B = j + 1$, we find

$$\delta r_j \simeq \frac{dr}{dP} [-\delta P_{j-1} + (2 + e) \, \delta P_j - \delta P_{j+1}],$$

$$\frac{dr}{dP} = \frac{[\alpha/\Delta z + K'\Delta t/((1 - \alpha) \, \rho_l \Delta z)]}{[u_{\mathrm{v}} + \Delta z/\Delta t + K'\Delta z/(\alpha \rho_{\mathrm{v}})]}, \tag{13}$$

and

$$e = \frac{\alpha}{\Delta t} \frac{d\rho_{\mathrm{v}}}{dP} \frac{dP}{dr} \equiv \frac{1}{c^2} \left(\frac{\Delta z}{\Delta t} \right)^2, \tag{14}$$

which we take to be the definition of the sonic velocity; note that for $\Delta t < \Delta z/u_{\mathrm{v}}$ and weak momentum coupling this gives

$$c^{-2} \simeq d\rho_{\mathrm{v}}/dP,$$

the equilibrium vapor sonic velocity, while for strong momentum coupling ($K' \rightarrow \infty$), unless Δt is very small,

$$c^{-2} \simeq (1 - \alpha) \, \rho_l/\rho_{\mathrm{v}} \cdot (d\rho_{\mathrm{v}}/dP),$$

which approximates the two-phase sonic velocity given by Henry *et al.* [11] (cf. [6]).

Equations (13)–(14) differ from the pressure increment rules in [6], in that we have retained the coupling of neighboring pressure increments, whereas the IMF technique use a one-point rule accounting only for $\delta r_j/\delta P_j$. Equation (14) shows that this approximation is acceptable as long as $\Delta t < \Delta z/c$. On that time scale, a change in pressure is largely taken up in the vapor compressibility. However, the restriction $\Delta t < \Delta z/c$ is just what we have been trying to avoid. What happens if $\Delta t \gg \Delta z/c$? Clearly, e rapidly becomes quite small, and the coupling of neighboring terms becomes more important.

To account for this, we use the coupled increment formula (13) and solve the system

$$\begin{bmatrix} 2+e_1 & -2 & 0 & 0 & \cdots & 0 \\ -1 & 2+e_2 & -1 & 0 & \cdots & 0 \\ 0 & -1 & 2+e_3 & -1 & \cdots & 0 \\ & & \vdots & & & \\ 0 & \cdots & & -1 & 2+e_{n-1} & -1 \\ 0 & \cdots & & 0 & 0 & 1+e_n \end{bmatrix} \begin{bmatrix} \delta P_1 \\ \delta P_2 \\ \delta P_3 \\ \vdots \\ \delta P_{n-1} \\ \delta P_n \end{bmatrix} = \begin{bmatrix} -(dP/dr)\, r_1 \\ -(dP/dr)\, r_2 \\ -(dP/dr)\, r_3 \\ \\ -(dP/dr)\, r_{n-1} \\ 0 \end{bmatrix} \tag{15}$$

by Gaussian elimination. (Applying the increment rule in [6] would be similar to approximating this matrix by its diagonal.) This is exactly the type of tridiagonal matrix one obtains in solving approximately the elliptic boundary value problem for one-dimensional diffusion. The diffusion length has been replaced here by its analog $e^{1/2} = \Delta z/(c\,\Delta t)$, which is the number of intervals traversed in time Δt by a signal propagating with velocity c. For illustration, we have chosen a left boundary condition of known velocities (prescribed inflow), which as a pressure boundary condition is analogous to reflection. At the right we have supposed a known pressure. As Δt becomes large, and hence $e \to 0$, the domain of influence of each point spreads, and the domain of strong interaction of each point in the implicit difference scheme (viz., the set of z_k such that $| z_k - z_j | < c\,\Delta t$) also spreads, and the implicit phase of the calculation becomes more elliptic in nature. This is natural since a parabolic problem (generated by an elliptic operator) can be regarded as the limiting case of a hyperbolic problem as $c \to \infty$, or conversely, as the time scale considered becomes much longer than the spatial dimensions of the problem divided by c. All this is known for single-fluid calculations, and we have merely generalized to the two-fluid case. To implement this coupled increment method, we precalculate dr/dP and e approximately. Since more than one iteration will usually be needed, the δP_j for a single iteration have no exact value, and any approximation which does not adversely affect the rate of convergence is permissible. Values for dr/dP and e calculated in the explicit phase work quite well, and we even limit the calculation to a few nodes and interpolate for the others. We also retain an over-/under-relaxation parameter, but this seldom has any real effect, since the asymptotic error reduction without acceleration (dominance ratio) for the coupled increment method is about an order of mangitude per iteration. This holds true for the examples below, where Δt is somewhat larger than $\Delta z/u_v$ and an order of magnitude or more greater than $\Delta z/c$. Rarely are more than a few iterations necessary.

As indicated above, measures must be taken to assure the numerical stability of the liquid flow. We have chosen an artificial diffusion of liquid mass in Eq. (9), whose magnitude is found from calculating certain truncation error terms according to a method of Hirt [10]. We use

$$\lambda = \left(\frac{1}{4}\frac{\partial u_l}{\partial z}\Delta z^2 + u_{l,\max}^2 \Delta t\right)((1-\alpha)\,\rho_l)^2 \frac{\partial^2 v_l'}{\partial z^2}$$

where $v_l' = ((1 - \alpha)\,\rho_l)^{-1}$ is chosen as the quantity apt to be linear in a stable flow. We also tried donor-cell differencing, but found the artificial diffusion method more satisfactory. The donor-acceptor technique (cf. [12]) has not been tried.

LIQUID FLOW INTERFACE

To use the XI method of two-phase flow calculation for sodium channel boiling transients requires a more complete, realistic model. Such transients start from a single-phase liquid flow regime in a long, narrow subchannel. With either a decreasing inlet flow rate or an increasing heat flux, liquid temperature rises and boiling may start at the end of the heated portion. Two-phase flow creates a larger pressure drop which tends to decrease inlet flow, accelerating growth of the two-phase region. Due to the low pressure, the two-phase region quickly develops high void fractions; correct calculation of the single-phase–two-phase interface(s) is essential to a realistic analysis.

Introducing this interface into the Eulerian mesh scheme of the XI method poses a problem. Adding a mesh point for the interface would create a cell of length $\Delta z'$ which might be very small, upsetting the stability criteria. Recently a smearing technique has been proposed for phase interfaces without calculating their location [12], but the two-fluid model was not used. We have decided to include the boiling front location, and to use a different, unconditionally stable method for the fractional cell. Both the liquid flow region and the fractional two-phase cell are solved using a directed method, which we shall discuss only briefly.

Initially the flow is liquid along the entire channel. A directed single-phase integration method assumes P, h_l, and $G = \rho_l u_l$ known at time t along the channel, and at $t + \Delta t$ at the inlet. Referring back to Fig. 1, the three difference equations of liquid mass, momentum, and energy span points A to B for spatial derivatives, and B and b for time differences. Velocities are edge-centered. These difference equations provide three nonlinear relations for the three unknowns P, h_l, u_l at B, which can be solved since the values at A and b are assumed known. Repeating the process, we advance one level at a time to the outlet. In case the outlet pressure is specified as a boundary condition, and, say, the inlet flow $G_I(t + \Delta t)$ is unknown, we use a shooting method: an estimate of G_I is used to start a trial integration, the resulting outlet pressure is compared to the desired value, the difference determines a correction to G_I, and the channel is integrated again until the error is small. Details are set forth in [13]; suffice it to say that with proper care the average number of repeat integrations per time step can be reduced practically to zero so the method is quite efficient.

Now suppose the liquid temperature rises to boiling, for simplicity at the outlet. Past the boiling point, the directed method becomes two phase; each trial integration is the same, but the last cell, once calculated as if it were all liquid, is then reexamined to find by interpolation the exact location of boiling. From this point to the outlet, a two-phase directed method does a new integration using the two-fluid model. Then shooting is applied as before.

The two-phase directed method is similar to the liquid directed method, except that each step now involves six finite difference equations supplying six nonlinear equations for six unknowns at B, for example P, α, u_l, u_v, h_l, and S. One procedure for solving these equations is given in [13], while an improved technique is set forth in [1].

There is another possible use for the directed method: one can try calculating all two-phase cells this way and suppress XI. This will work provided the two-phase region does not become too large; it amounts to an extension of FLINA [4] to the two-fluid model, and provides and independent check in that the same physical model can be calculated with two different numerical methods. Our experience shows the agreement is extremely good.

Much additional work is required in order to supply a realistic context for the sodium channel boiling calculation. Aspects already in the computer code, called ALECTO, include a simple treatment of conduction from the fuel pin (to give q), and calculation of flow reversal at the inlet. Needed are a detailed conduction calculation, treatment of the single-phase vapor flow region after dryout, more experience in comparing with experiments, and inclusion in a transient reactor analysis system code. An extension to handle several subchannels connected by transverse flow might be another interesting development.

EXAMPLES

As mentioned above, a goal of the present work has been to develop methods capable of treating a wide range of boiling transients. Some examples chosen here demonstrate this capacity.

An example of a very rapid transient is the propagation of a small pressure pulse along a channel with two-phase flow. Although not directly of great interest for reactor accident analysis, the test is theoretically important to show that the XI method has no lower limit on the time step, and hence no upper limit on the length of two-phase flow which can be considered. Directed methods with shooting probably could not carry out such a calculation; our directed method alone certainly failed.

The pressure pulse problem illustrated in Fig. 2 begins from a steady-state two-phase channel flow of sodium at atmospheric pressure, with heat flux from the surrounding wall. Void fractions are large ($< 80\ \%$) and the interphase friction is such that u_v is about 10 times u_l. (This steady state was actually the result of a preceding transient calculation during which the inlet flow and outlet pressure were fixed in mid-transient, and the calculation was pursued to $t = 50$ sec, at which time the flow had become almost completely steady). The boundary conditions are inlet flow (α, u_v, u_l, h_l at left) and outlet pressure (P at right). Figure 2 shows pressure profiles along the channel, relative to the initial profile, for several instants following the introduction of a 1-mbar pressure pulse at the right. The time step between profiles is about equal to $\Delta z/c$, where c calculated from Eqs. (13)–(14) is about 600 m/sec, very close to $(d\rho_v/dP)^{1/2}$. (In the actual calculation there were more than one time step between the profiles illustrated). As can be seen, the pulse travels at the

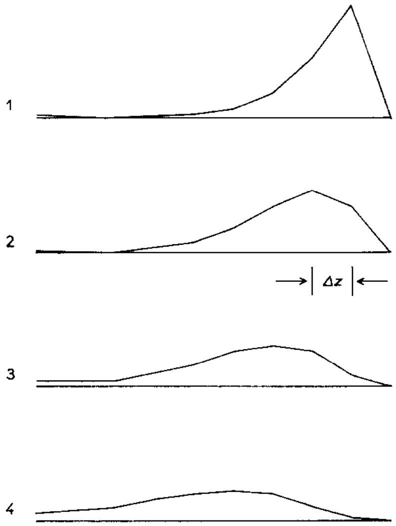

FIG. 2. Calculated axial profiles of pressure disturbance for times 1(1) $4\Delta t$ after introducing a pulse on the right; $\Delta t = \Delta z/c$.

expected rate of about one spatial cell per time step, with strong dissipation (at least part of which is numerical due to the implicit differencing).

Our second example concerns a very slow transient, corresponding to a supposed loss of power to the coolant pumping system of a sodium-cooled fast reactor. The pumping system is supposed to have a large mechanical inertia. Out-of-pile experiments have simulated thermohydraulic conditions for such an accident in a single subchannel using an electrically heated forced convection loop (Schmitt [14]). Figure 3 shows void fraction profiles at various times calculated for one of these out-of-

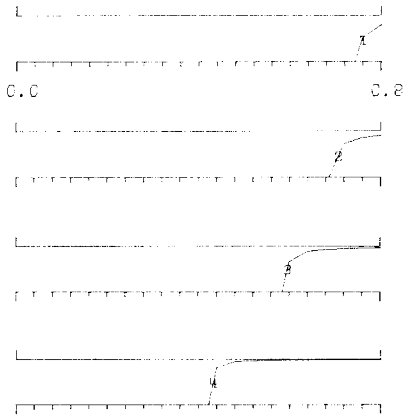

FIG. 3. Calculated axial profiles of void fraction at times $t = 8(4)\ 20$ sec after onset of boiling for a slow boiling transient in 0.8-m-long electrically heated subchannel; in frame 4, net inlet liquid flow rate is low but still positive.

pile experimental runs, 327 Tr1, using a 0.8-m-long channel heated along its entire length. The transient began with the outlet very near boiling and was calculated to $t = 30$ sec, by which time the (average) inlet flow fell to less than 0.1 m/sec (from 0.68 m/sec initially) and the void fraction at the outlet exceeded 99 %. (The calculation agreed well with available experimental data, but we repeat that much more serious empirical evaluation of K and F is badly needed.) At this point in the actual experiment the liquid present near the outlet became insufficient to wet the heated wall of the channel, and the flow entered a "chugging" phase of periodic dry-out and rewetting of the wall, for which the appropriate model has not yet been included in the code. It is clear from the calculation that for such a gradual transient dry-out will start near the outlet, that is near the end of the heated length.

Since this transient evolves slowly, long time steps are desirable. The results in Fig. 3 were obtained using $\Delta t = 2\,\Delta z/u_{\mathrm{v,max}}$ (the maximum consistent with the restriction due to $u_{l,\max}$ for this problem); a calculation using $\Delta t = \Delta z/u_{\mathrm{v,max}}$ differed by a 0.05 sec. offset at the end of 20 sec. The latter calculation took virtually twice as long as the former, indicating that computational effort per time step was the same in each case.

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